Birzeit University
Department of Physics
Quantum Mechanics Phys635
Fall 2018
First Exam, Oct. 30th 2018

1. Let K be the operator defined by $K = |\phi><\psi|$, where $|\phi>$ and $|\psi>$ are two vectors of the state space.
   (a) (5 points) Under what conditions K is Hermitian?
   (b) (5 points) Calculate K^2, under what condition K is projector.
   (c) (5 points) Show that K can always be written in the form $K = \lambda P_1 P_2$, where λ is a constant to be calculated and P_1 and P_2 are projectors.

2. (21 points) Consider the following Hamiltonian:

$$H = -(\frac{eB}{mc})S_z$$

   Write the Heisenberg equation of motion for S_x, S_y and S_z and obtain the expectation value as a function of time.

3. (10 points) Show that for harmonic oscillator that the eigenvalues of the number operator $\hat{N}$ are positive integers or zero.

4. The ammonia molecule NH_3 has two different possible configurations: One (which we will call $|1>$), where the nitrogen atom is located above the plane spanned by the three H atoms, and the other one (which we will call $|2>$) where it is below. These two states span the Hilbert space. In both states, the expectation value of the energy $<n|H|n>$ is the same, E (n = 1 , 2). On the other hand, the two states are not eigenstates of the Hamiltonian; in fact, we have $<2|H|1>=<1|H|2>=-V$ (where V is some positive number).
   (a) (6 points) Write down the Hamiltonian as an outer product of the states $|1>$ and $|2>$
   (b) (6 points) Find the eigenvalues and eigenvectors of the Hamiltonian.
   (c) (5 points) What is the probability to find nitrogen atom above or below in the two states?
   (d) (5 points) Consider the parity operator in which all coordinates change sign. Find the eigenvalues and eigenvectors of the parity operator.

5. Consider a particle of mass m and charge q in one dimensional harmonic oscillator, we can define the following operator:

$$U(\lambda) = e^{\lambda(a-a^\dagger)} = e^{-\lambda a}e^{-\lambda a^\dagger}e^{\lambda^2/2}$$
$$\tilde{H} = U(\lambda)HU^\dagger(\lambda)$$
$$\tilde{a} = U(\lambda)aU^\dagger(\lambda)$$
$$\tilde{a^\dagger} = U(\lambda)a^\dagger U^\dagger(\lambda)$$

   (a) (5 points) Show that $\tilde{a} = a - \lambda$
   (b) (5 points) Show that $\tilde{a^\dagger} = a^\dagger - \lambda$
   (c) (5 points) Write $\tilde{H}$ in terms of a, $a^\dagger$ and λ
   (d) (8 points) If the harmonic oscillator is subjected to an external E uniform electric field, Show that $\lambda = \frac{qE}{\omega}\sqrt{\frac{1}{2m\hbar\omega}}$

Good Luck

☺

Question:	1	2	3	4	5	Total
Points:	15	21	10	22	23	91
Score:						